TRINITY COLLEGE

Mathematics Applications Units 3, 4
Test 5 2019

Calculator Assumed
Finance

STUDENT'S NAME Solutions

DATE: Wednesday 14th August **TIME:** 50 minutes **MARKS**: 49

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Clifford has a sum of money to invest for a period of time without touching the investment. He is currently deciding between the two options below. Which investment should Clifford choose? Show mathematical justification to support your answer.

Option A: Invest the money in an account earning compound interest at a rate of 5.38% per annum with interest compounding daily

Option B: Invest the money in an account earning compound interest at a rate of 5.5% per annum with interest compounding annually

A - Effective interest rate = 5.5269% pa. ✓

B - Effective interest rate = 5.5% p.a. ✓

Option A is better as it has a higher effective interest rate. ✓

2. (6 marks)

The Trinity mathematics department initially purchased a photocopier for \$3800. After two years, the value of the photocopier had depreciated to \$3150.

(a) Assuming flat rate depreciation, calculate the amount the photocopier depreciates each year. [1]

$\frac{3800 - 3150}{2} = \$325$ /year

(b) If the photocopier continues to depreciate at the same rate, how many years after being purchased will value of the photocopier be \$550? [1]

$3800 - 325x = 550$

$x = 10$

$\therefore$ 10 years

A sound system was initially purchased by the Trinity music department for \$2100. After five years, the value of the system had depreciated to \$1040.

(c) Assuming reducing balance depreciation, calculate the annual percentage rate by which the value of this sound system depreciated. Express your answer correct to two decimal places. [2]

$2100 \times x^5 = 1040$

$x = 0.8689$ ✓

$\therefore$ Depr = 13.11 % ✓

(d) If the sound system is written off when the value reaches 15% of its purchase price, what is the effective life of the sound system? [2]

$2100 \times 0.15 = 315$

$2100 \times 0.8689^x = 315$ ✓

$x = 13.498$

$\therefore$ 14 years ✓

3. (10 marks)

James would like to buy a new house and decides to take out a mortgage for $550 000 at a fixed rate of 5.24% per annum, compounding monthly.

(a) How much interest will James pay in the first month? [2]

$$550\,000 \times \frac{5.24}{100 \times 12} = \$\,2401.67$$ ✓ ✓

James pays $3 150 at the end of each month, after interest has been added.

(b) Write the recursive rule for the value James' mortgage at the end of each month. [2]

$$T_{n+1} = T_n\left(1 + \frac{5.24}{100 \times 12}\right) - 3150$$ ✓

$$T_0 = 550\,000$$ ✓

(c) What is owing on the mortgage after 12 months? [1]

$$T_{12} = 540\,801.16$$

(d) How much interest has been paid on the loan after 12 months? [2]

$$12 \times 3150 - (550\,000 - 540\,801.16)$$ ✓

$$= \$28\,601.16$$ ✓

(e) How much money would James have saved after 10 years if he had made monthly payments of $3 250 per month instead of $3 150? [3]

10yrs at 3150 = 432 292.79 ✓

10yrs at 3250 = 416 563.54 ✓

Saving = $15 729.25 ✓

4. (8 marks)

Timothy is saving to purchase a new car worth \$7 000. He invests \$1 500 into an account which will earn interest at a rate of 3.6% per annum compounded monthly. At the end of each month, after the interest has been added, he will make an additional deposit of \$200.

The table below shows the progress of the investment for the first four months.

Month	Value at beginning of month	Interest for the month	Deposit	Value at end of month
1	1500.00	4.50	200.00	1704.50
2	1704.50	5.11	200.00	1909.61
3	1909.61	5.73	200.00	2115.34
4	2115.34	***A***	200.00	***B***

(a) Determine the value of ***A*** and ***B***. [2]

$A = 6 \cdot 35$ ✓

$B = 2321 \cdot 69$ ✓

(b) Write a recursive rule to represent the value of the investment at the end of each month. [2]

$T_{n+1} = T_n\left(1 + \frac{3 \cdot 6}{100 \times 12}\right) + 200$ ✓

$T_0 = 1500$ ✓

(c) Determine the length of time it will take Timothy to save enough to purchase the car. [1]

26 months

(2 yrs, 2 months)

(d) How much interest has Timothy's investment accrued by the end of one year? [2]

$T_{12} = 3994.90$

$(3994.90 - 1500) - (200 \times 12)$ ✓

$= \$ 94.90$ ✓

(e) Determine the amount that Timothy would need to deposit each month in order to save the $7 000 in just 18 months. [1]

$\$293.34$

5. (2 marks)

Laura decides to invest $15 000 into an account that will earn 7.6% interest compounded daily. If she is to withdraw $1000 at the end of each year, how long will it take for her investment to reach a balance of $0? Explain your answer.

✓ Won't reach $0

✓ Withdrawing less than interest earned.

6. (7 marks)

Robert sets up his pension fund on 1st July 2018 with a principal of $850 000. The fund guarantees an annual growth rate of 7.5% per annum compounded monthly and he plans to take an annuity of $75 000 each year on 1st July, starting in 2019.

(a) Calculate the balance in the fund after the annuity is withdrawn in July 2022. [2]

N = 4
I = 7.5
PV = -850 000
PMT = 75000 ✓
P/Y = 1
C/Y = 12

FV = 809 531.47 ✓

The investment fund revised its annual interest rate to 9% compounded monthly on 1st July 2022 guaranteed for the period to July 2027 and Robert continued withdrawing $75 000 as usual.

(b) Calculate the balance in the fund after a withdrawal is made on 1st July 2027. [2]

N = 5
I = 9
PV = -809 531.47
PMT = 75000 ✓
P/Y = 1
C/Y = 12

FV = 815 197.73 ✓

(c) Calculate to the nearest $100, the maximum amount Robert could withdraw annually, starting in 2022, without decreasing his balance. [2]

PV = -809 531.47
FV = 809 531.47 ✓
P/Y = 1
C/Y = 12

N = Any value
I = 9 ✓

PMT = 75 900 (to nearest 100)

(d) What name is given to the type of investment in part (c)? [1]

Perpetuity

7. (13 marks)

Daniel takes out a \$16 500 loan to purchase a car after paying a \$1 300 deposit. The car dealer offered the loan at an introductory interest rate of 1.95% p.a. for the first year and then the rate becomes 3.45% p.a. for the remaining time of the loan. Interest is added monthly and Daniel has calculated he can afford to make monthly repayments of \$450.

(a) (i) Express the loan repayment process for the first year as a recursive formula. [2]

$T_{n+1} = T_n \left(1 + \frac{1.95}{100 \times 12}\right) - 450$ ✓

$T_0 = 16\,500$ ✓

(ii) How much does Daniel still owe after one year? [1]

$T_{12} = \$11\,376.12$

(b) How much does Daniel owe after two years? [3]

N = 12 P/Y = 12

I = 3.45 C/Y = 12 FV = 17261.07 ✓

PV = -11376.12 ✓✓

PMT = -450

(c) How long does it take Daniel to repay the loan? [2]

12 months + 27 months ✓

= 39 months ✓ (3 years, 3 months)

(d) Determine the amount of the final repayment. [2]

39th month = -322.18 ✓

450 - 322.18 = \$127.82 ✓

(e) Calculate the total cost of the car. [3]

$(38 \times 450) + 127.82 + 1300$ ✓✓

$= \$18\,527.82$ ✓